MISCELLANEA.

CONTENTS:

This Paper, by Professor Jevons, was read in Section F of the British Association, in 1862, but has not been hitherto printed.

I.—*Brief Account of a General Mathematical Theory of Political Economy.*

1. The following paper briefly describes the nature of a Theory of Economy which will reduce the main problem of this science to a mathematical form. Economy, indeed, being concerned with quantities, has always of necessity been mathematical in its subject, but the strict and general statement, and the easy comprehension of its quantitative laws has been prevented by a neglect of those powerful methods of expression which have been applied to most other sciences with so much success. It is not to be supposed, however, that because economy becomes mathematical in form, it will, therefore, become a matter of rigorous calculation. Its mathematical principles may become formal and certain, while its individual data remain as inexact as ever.

2. A true theory of economy can only be attained by going back to the great springs of human action—*the feelings of pleasure and pain.* A large part of such feelings arise periodically from the ordinary wants and desires of body or mind, and from the painful exertion we are continually prompted to undergo that we may satisfy our wants.

Economy investigates the relations of ordinary pleasures and pains thus arising, and it has a wide enough field of inquiry. But economy does not treat of all human motives. There are motives nearly always present with us, arising from conscience, compassion, or from some moral or religious source, which economy cannot and does not pretend to treat. These will remain to us as outstanding and disturbing forces; they must be treated, if at all, by other appropriate branches of knowledge.

3. We always treat feelings as being capable of *more or less*, and I now hold that they are quantities capable of scientific treatment.

Our estimation of the comparative amounts of feeling is performed in the act of choice or volition. Our choice of one course out of two or more proves that, in our estimation, this course promises the greatest balance of pleasure. When there is a large overbalancing force on one side, indeed, the estimation of the amount of this balance is no doubt very rude; but all the critical points of the theory will depend on that nice estimation of the opposing motives which we make when these are nearly equal, and we hesitate between them.

4. As several writers have previously remarked, feelings have two dimensions, *intension* and *duration*. A pleasure or a pain may be either weak or intense in any indivisible moment; it may also last a long or a short time. If the intensity remain uniform, the quantity of feeling generated is found by multiplying the units of intensity into the units of duration. But if the intensity, as is usually the case, varies as some function of the time, the quantity of feeling is got by infinitesimal summation or *integration*.

Thus, if the duration of a feeling be represented by the abscissa of a curve, the intensity will be the ordinate, and the quantity of feeling will be the area.

5. Pleasure and pain, of course, are opposed as positive and negative quantities.

6. A principle of the mind which any true theory must take into account is that of *foresight*. Every expected future pleasure or pain affects us with similar feelings in the present time, but with an intensity diminished in some proportion to its uncertainty and its remoteness in time. But the effects of *foresight* merely complicate without altering the other parts of the theory.

7. Such are the main principles of feeling on which economy is founded. A second part of the theory proceeds from feelings to the *useful objects* or *utilities* by which pleasurable feeling is increased or pain removed.

An object is useful when it either affects the senses pleasurably in the present moment, or when, by foresight, it is expected that it will do so at some future time. Thus we must carefully distinguish *actual utility* in present use from *estimated future utility*, which yet, by allowing for the imperfect force of anticipation, and for the uncertainty of future events, gives a certain present utility.

8. *Amount of utility* corresponds to amount of pleasure produced. But the continued uniform application of an useful object to the senses or the desires, will not commonly produce uniform amounts of pleasure. Every appetite or sense is more or less rapidly satiated. A certain quantity of an object received, a further quantity is indifferent to us, or may even excite disgust. Every successive application will commonly excite the feelings less intensely than the previous application. The utility of the last supply of an object, then, usually decreases in some proportion, or as some function of the whole quantity received. This variation theoretically existing even in the smallest quantities, we must recede to infinitesimals, and what we shall call the *coefficient of utility*, is the ratio between the last increment or infinitely small supply of the object, and the increment of pleasure which it occasions, both, of course, estimated in their appropriate units.

9. The coefficient of utility is, then, some generally diminishing function of the whole quantity of the object consumed. Here is the most important law of the whole theory.

This function of utility is peculiar to each kind of object, and more or less to each individual. Thus, the appetite for dry bread is much more rapidly satisfied than that for wine, for clothes, for handsome furniture, for works of art, or, finally, for money. And every one has his own peculiar tastes in which he is nearly insatiable.

10. A third part of the theory now treats of *labour;* which, although the means by which we seek pleasure, is always accompanied by a certain painful exertion, rapidly increasing as some function of the intensity or the duration of the labour. Thus, labour will be exerted both in intensity and duration until a further increment will be more painful than the increment of produce thereby obtained is pleasurable. Here labour will stop, but up to this point it will always be accompanied by an excess of pleasure.

It is obvious that the final point of labour will depend upon the final ratio of utility of the object produced.

11. I assume, as obviously true, that the abilities of men are infinitely varied, whether by nature or by education, so that both the same person may vary in his power of producing different objects, and any two persons may vary in respect of the same object.

This, indeed, is in direct opposition to the erroneous simplification of the science effected by Ricardo, when he assumed that all labourers have a certain uniform power; the higher classes of mechanics and other skilled or learned producers being treated as mere exceptions to the rule.

12. The theory of rent, which here comes in, is not materially different from that of Dr. Anderson and later writers.

13. We now arrive at the *theory of exchange*, which is a deduction from the laws of utility.

If a person has any useful object, but an object belonging to another person would have greater utility, he will be glad to give the one in return for the other. But it is a necessary condition that the other person will likewise gain, or at least not lose by the exchange.

Whether the exchange will take place or not can only be ascertained by estimating the utility of the objects on either side, which is done by integrating the appropriate functions of utility up to the quantity of each object as limits. A balance of utility on both sides will lead to an exchange.

14. Suppose, however, that the useful objects on either side are commodities of which more or less may be given, and this even down to infinitely small quantities. Such is substantially the case in ordinary commercial sales. There are now no definite amounts of utility to be balanced against each other, but the one person will now give to the other so much of his commodity, and at such a ratio of exchange, that if he gave an infinitely small quantity, either more or less, but at the same rate, he would not gain in utility by it. The increments of utility lost and gained at the limits of the quantities exchanged must be equal, otherwise further exchange would take place.

The ratio of the increments of the commodities, however, would be indeterminate but for the existence of a law that all quantities of the same commodity, being uniform in kind, must be exchanged at the same rate. The last increments, then, must be exchanged, in the ratio of the whole quantities exchanged. To explain in ordinary words how the adjustment takes place under this condition is almost impossible. But light is at once thrown on the whole matter by stating that in every such exchange we have two *unknown quantities* and two equations by which to determine them. The *unknown*

quantities are the quantities of commodity given and received. The known quantities are those of the commodities previously possessed. We have also the functions of utility of the commodities with the respect to the persons. An equation may thus be established on either side between the utility gained and sacrificed at the ratio of exchange of the whole commodities, upon the last increments exchanged.

15. When the useful object on one side only is infinitely divisible, we shall have only one unknown quantity, namely, that of the divisible commodity given for the indivisible object, and also one equation to determine it by, namely, that on the part of the person holding the divisible commodity, and able to give more or less for it. But this does not apply to unique objects, like a statue, a rare book, or gem, which do not admit of the conception of more or less.

When both commodities are indivisible as first supposed (section 13), we have neither unknown quantities nor equations.

16. The equations in an exchange may prove impossible, or without solutions. This will indicate either that no exchange of commodity can take place at all, or that at least one of the parties to the exchange is not satisfied even with the whole of the commodity formerly belonging to the other.

17. The principle of exchange thus deduced in the case of two persons and two commodities, applies to any number of persons and commodities. It, therefore, applies not only to the general inland trade of a country, but to the trade between aggregates of men or nations—international trade.

The number of equations is very rapidly increased according to the simple law of combinations.

18. Of course such equations as are here spoken of are merely theoretical. Such complicated laws as those of economy cannot be accurately traced in individual cases. Their operation can only be detected in aggregates and by the method of averages. *We must think under the forms of these laws in their theoretic perfection and complication; in practice we must be content with approximate and empirical laws.*

19. Let it be remarked, that though the exchanges be regulated by equations, there cannot be equality in the whole utilities gained and lost, which are found by integrating the functions of utility of the respective commodities before and after exchange. The balance is the gain of utility, and from the nature of exchange there must be a gain on one side at least.

20. Combining the theory of exchanges with that of labour and production, the quantity which each person produces will be dependant upon the result of the exchanges; for this may greatly modify the conditions of utility.

A new set of unknown quantities are thus introduced; but it will be found that just as many new equations to determine them may be established. Each such equation is between the utility of the last increment of produce and the increment of labour necessary to produce it.

21. The only further part of the theory which I will here at all attempt to explain is that referring to capital. I shall give a

definition of capital different from the established one, and much simpler. Mr. J. S. Mill says (*Principles*, 3rd edition, vol. i, p. 67), "What capital does for production is to afford the shelter, protection, tools and materials which the work requires, and to feed and otherwise maintain the labourers during the process."

To understand capital properly, we must omit all but the last enumerated part. Thus, I define capital as consisting of *all useful objects which, in supplying a labourer's ordinary wants and desires, enable him to undertake works of which the result will be deferred for a greater or less space of time.* Capital, in short, is nothing but *maintenance of labourers.*

It is, of course, perfectly true that buildings, tools, materials, &c., are a necessary means of production; but they are already the product of labour assisted by capital or maintenance. They are the results of the application of capital to labour at an imperfect stage.

Without capital a person must have immediate returns, or else he perishes. With capital he may sow in the spring that he may reap in the autumn; or he may undertake labour-saving enterprises, such as roads and railways, which will not make a full return for many years. Most improved modes of applying labour require that the enjoyment of the result shall be deferred.

22. While amount of capital is estimated by the amount of utility of which the enjoyment is deferred, *amount of employment of capital* is the amount of utility multiplied by the number of units of time during which its enjoyment is deferred.

23. The interest of all capital in a market is of one rate only, and that, therefore, the lowest rate; because capital consists only in maintenance, and may therefore be applied indifferently to any branch of industry. Buildings, tools, &c., which have hitherto been classed with capital, are, on the contrary, usually applicable only to the single purpose for which they were designed. The profit they bring, therefore, in no way follows the laws of the interest of capital, but rather those of rent, or the produce of natural agents. This has been already remarked by Professor Newman, in his *Lectures on Political Economy*, and by other writers.

24. As labour must be supposed to be aided with some capital, the rate of interest is always determined by the *ratio which a new increment of produce bears to the increment of capital by which it was produced.* As the interest of all capital must be uniform, the benefit which the mass of capital already available confers upon the labourer goes for nothing in determining the rate of interest, which depends solely upon the portion last added, or which may be added.

25. We can now easily explain the known fact, that the interest of capital always tends to fall very rapidly as its amount increases, in proportion to the labour it supports. It is because for equal increments of time the necessary increments of capital increase with the time. Thus, if I undertake a work which I can finish in one year, I have to await the result on an average only half a-year. If, however, I work a second year before getting the result, I wait a whole year for the former year's work and half a-year for the second year's work. Thus I employ at least three times as much capital in the second year as in the first. In the third year I should

employ at least five times as much capital, in the fourth year at least seven times, and so on. Unless, then, the advantages of the successive deferments increase in the arithmetical series 3, 5, 7, 9, &c., the proportional profit from the new additions must fall, and, as was said before, the lowest rate for which capital may be had governs the rate of all other capital.

26. It is the accepted opinion of writers of the present day, that the rate of interest tends to fall because the soil does not yield proportionate returns as its cultivation is pushed. But I must hold that this decrease in the proportionate returns would chiefly fall upon the wages of the labourer. The interest of capital has no relation to the absolute returns to labour, but only to the increased return which the last increment of capital allows.

27. Having thus explained some of the principal features of the theory, I shall close without venturing into the higher complications of the subject, where the effects of money, of credit, of combination of labour, of the risk or uncertainty of undertakings, and of bankruptcy, are taken into account.

The last result of the theory will be to give a determination of the rate of wages, or the produce of labour after deduction of rent, interest, profit, insurance and taxation, which are so many payments which the labourer makes for advantages enjoyed.

II.—*Statistics of Live Stock in the United Kingdom.*

THE Statistical Department of the Board of Trade have recently published the first complete return of live stock in the United Kingdom of Great Britain and Ireland; the accounts heretofore obtained have related to the latter country only. Some of the most important items are set out in the subjoined tables:—

I.—*Total Number of Cattle, Sheep, and Pigs in each Division of the United Kingdom.*

Divisions of United Kingdom.	Date of Returns.	Number of Live Stock.				
		Cattle.			Sheep.*	Pigs.
		Cows.	Other Cattle.	Total.		
England	5th March, 1866	1,290,529	2,016,505	3,307,034	15,124,541	2,066,299
Wales	"	222,546	318,855	541,401	1,668,663	191,604
Scotland	"	370,457	566,954	937,411	5,255,077	219,716
Ireland	Year 1865	1,386,176	2,107,238	3,493,414	3,688,742	1,299,893
Isle of Man	5th March, 1866	7,755	10,932	18,687	55,954	10,956
Channel Islands:—						
Jersey	"	5,815	6,222	12,037	517	6,332
Guernsey, &c.	"	3,030	3,946	6,976	1,214	5,599
Total for United Kingdom	—	3,286,308	5,030,652	8,316,960	25,794,708	3,800,399

* The number of sheep in these returns, so far as they relate to the northern counties of England and to Scotland, is probably short by a large number of the lambs of the present year in consequence of the date at which the returns were made.

II.—*Number of each kind of Live Stock, and Number of Cattle Died of, or Killed on Account of Plague, in each County of Great Britain.*

Counties (Proper) in Great Britain.	Number of Live Stock Existing upon 5th March, 1866.			Estimated Ordinary Stock of Cattle.	Number of Cattle Died of, or Killed on Account of Plague, up to Week ended 21st April, 1866.	Percentage of Losses of Cattle from Plague up to 21st April, 1866, computed upon Estimated Ordinary Stock.
	Cattle.	Sheep.	Pigs.			
ENGLAND.						
Bedford	25,513	180,250	36,796	25,915	442	1·70
Berks	30,149	327,316	41,265	30,632	499	1·62
Buckingham	51,840	263,015	39,266	52,684	1,856	3·52
Cambridge	31,731	255,036	58,225	37,086	6,640	17·90
Chester	93,044	96,989	57,692	125,192	42,922	34·28
Cornwall	133,652	300,049	63,920	134,470	926	·69
Cumberland	109,225	396,021	40,742	112,377	5,370	4·78
Derby	113,195	176,122	31,452	113,569	1,029	·90
Devon	184,077	769,126	94,475	184,203	169	·09
Dorset	70,002	492,623	38,676	70,014	15	·02
Durham	52,322	146,696	14,140	52,695	406	·77
Essex (extra metropolitan)*	50,736	373,129	98,948	52,582	1,928	3·66
Gloucester	96,831	356,373	50,933	96,907	116	·12
Hants	48,690	619,598	79,789	48,976	299	·61
Hereford	65,184	257,196	25,260	65,717	548	·83
Hertford (extra metro.)*	20,830	206,858	32,194	20,166	486	2·29
Huntingdon	17,667	117,821	26,602	19,804	2,238	11·30
Kent (extra metropolitan)*	50,835	709,056	66,763	52,197	1,371	2·63
Lancaster	202,552	217,615	50,375	205,130	3,841	1·87
Leicester	89,115	290,554	23,250	89,355	333	·37
Lincoln	169,294	1,088,204	91,522	175,654	9,955	5·66
Monmouth	36,735	131,158	13,152	36,735	—	—
Norfolk	92,386	596,683	115,876	96,960	5,638	5·81
Northampton	74,262	435,837	38,366	76,358	2,476	3·24
Northumberland	78,431	635,487	24,621	79,319	919	1·16
Nottingham	67,165	245,532	29,083	68,386	2,308	3·37
Oxford	42,135	333,304	42,372	43,447	2,566	5·90
Rutland	11,651	75,755	3,754	11,669	19	·16
Salop	107,208	327,612	59,674	110,711	5,412	4·89
Somerset	173,547	636,975	75,469	173,659	178	·10
Stafford	107,298	231,936	47,967	109,806	3,585	3·26
Suffolk	75,767	407,929	133,498	57,359	2,022	3·52
Surrey (extra metropolitan)*	22,037	121,928	23,618	23,179	1,171	5·04
Sussex	74,670	485,056	42,642	75,734	1,066	1·40
Warwick	67,249	285,878	36,613	67,817	598	·88
Westmoreland	55,328	224,664	7,413	55,528	—	—
Wilts	77,724	596,822	61,012	77,823	99	·13
Worcester	45,789	204,154	36,686	45,909	197	·43
York, East Riding	64,809	416,853	52,589	393,421	24,796	6·30
„ city of York						
„ North Riding	119,233	462,038	53,472			
„ West „	189,341	500,196	72,048			
Total for counties of England	3,269,247	14,995,444	2,032,210	3,369,953	134,486	4·00
Total for metropolitan police district	37,787	129,097	34,089	44,483	7,028*	15·80
Total for England	3,307,034	15,124,541	2,066,299	3,414,436	141,514	4·14

* This total of 7,028 includes 6,908 for the "metropolitan police district" up to the 31st of March, and 120 for the "metropolis," or districts under the Board of Works, from 31st of March to 21st of April.

II.—*Number of each kind of Live Stock, and Number of Cattle Died or Killed, &c.—Contd.*

Counties (Proper) in Great Britain.	Number of Live Stock Existing upon 5th March, 1866.			Estimated Ordinary Stock of Cattle.	Number of Cattle Died of, or Killed on Account of Plague up to Week ended 21st April, 1866.	Percentage of Losses of Cattle from Plague up to 21st April, 1866, computed upon Estimated Ordinary Stock.
	Cattle.	Sheep.	Pigs.			
WALES.						
Anglesea	35,427	33,715	17,875	35,427	—	—
Brecon	29,604	212,515	7,367	29,604	—	—
Cardigan	47,384	198,546	16,708	47,384	—	—
Carmarthen	84,106	110,295	19,339	84,106	—	—
Carnarvon	44,072	139,317	18,940	44,072	—	—
Denbigh	46,695	150,565	26,529	49,994	3,552	7·16
Flint	19,383	35,106	14,860	21,917	3,186	14·54
Glamorgan	45,911	177,484	14,335	45,911	—	—
Merioneth	33,343	235,091	7,703	33,343	—	—
Montgomery	58,628	220,241	20,863	58,628	—	—
Pembroke	68,842	64,412	21,739	68,842	—	—
Radnor	28,006	181,376	5,346	28,006	—	—
Total for Wales	541,401	1,668,663	191,604	547,234	6,738	1·23
SCOTLAND.						
Aberdeen	133,451	109,292	14,763	133,798	358	·27
Argyle	57,831	700,621	5,702	57,831	—	—
Ayr	75,544	262,973	13,502	76,281	781	1·02
Banff	36,542	55,409	6,171	36,542	—	—
Berwick	15,192	193,288	8,344	15,429	237	1·54
Bute	8,252	34,318	1,250	8,252	—	—
Caithness	19,999	82,676	3,012	19,999	—	—
Clackmannan	2,104	9,468	1,261	2,361	275	11·64
Dumbarton	10,564	53,405	1,196	11,287	785	6·96
Dumfries	44,364	371,486	18,612	45,067	717	1·59
Edinburgh	13,013	113,479	9,609	14,088	1,077	7·64
Elgin or Moray	20,406	69,078	5,772	20,406	—	—
Fife	27,297	93,685	14,193	30,776	3,781	12·28
Forfar	28,645	156,653	14,868	37,501	9,452	25·20
Haddington	9,659	91,414	7,646	10,170	511	5·02
Inverness	45,334	522,006	4,588	45,335	1	—
Kincardine	21,529	41,073	4,769	22,734	1,691	7·44
Kinross	4,028	22,450	1,286	4,596	660	14·38
Kirkcudbright	34,658	271,467	10,289	34,692	34	·10
Lanark	56,206	160,014	8,992	60,539	4,484	7·40
Linlithgow	8,029	23,070	3,166	8,704	696	8·00
Nairn	5,232	19,862	1,406	5,232	—	—
Orkney and Shetland	43,308	76,624	7,313	43,308	—	—
Peebles	5,970	125,831	1,220	5,975	5	·08
Perth	66,150	494,635	17,782	69,944	4,342	6·21
Renfrew	21,513	26,503	2,354	21,968	519	2·36
Ross and Cromarty	36,109	293,754	8,489	36,109	—	—
Roxburgh	16,084	310,537	6,094	16,149	65	·40
Selkirk	2,027	100,885	663	2,042	15	·73
Stirling	21,396	86,392	3,673	23,891	2,705	11·32
Sutherland	11,262	164,060	1,458	11,262	—	—
Wigton	35,703	118,669	10,273	35,703	—	—
Total for Scotland	937,411	5,255,077	219,716	967,975	33,191	3·43
Total for Great Britain	4,785,846	22,048,281	2,477,619	4,929,645	181,443	3·68

It appears from the same publication that there were on the 5th March, 1866, in England and Scotland 4,785,846 cattle, of which 1,883,532 were cows, 1,298,514 were "other cattle" aged two years and over; and 1,603,800 under two years of age; and that there were 22,048,281 sheep, of which 15,538,444 were one year old and above; and 6,509,837 under one year.

The next table exhibits the number of live stock in the United Kingdom and various foreign countries:—

III.—*Statement of the Population and Number of Live Stock in the under-mentioned Countries, according to the Latest Returns.*

Countries.	Date of Returns of Live Stock.	Population according to Latest Returns.	Cattle.			Sheep.	Pigs.
			Cows.	Other Cattle.	Total.		
United Kingdom	1865–66	29,070,932	3,286,308	5,030,652	8,316,960	25,795,708	3,802,399
Russia	'59–63	74,139,394	—	—	25,444,000	45,130,800	10,097,000
Denmark Proper	1861	1,662,734	756,834	361,940	1,118,774	1,751,950	300,928
Sleswig	'61	421,486	217,751	172,250	390,001	362,219	87,867
Holstein	'61	561,831	198,310	92,062	290,372	165,344	82,398
Sweden	'60	3,859,728	1,112,944	803,714	1,916,658	1,644,156	457,981
Prussia	'62	18,491,220	3,382,703	2,251,797	5,634,500	17,428,017	2,709,709
Hanover	'61	1,880,070	—	—	949,179	2,211,927	554,056
Saxony	'61	2,225,240	411,563	226,897	638,460	371,989	270,462
Wurtemberg	'61	1,720,708	466,758	490,414	957,172	683,842	216,965
Grand Duchy of Baden	'61	1,429,199	348,418	273,068	621,486	177,322	307,198
Grand Duchy of Hesse	'63	853,315	187,442	129,211	316,653	231,787	195,596
Grand Duchy of Nassau	'64	468,311	116,421	84,224	200,645	152,584	65,979
Grand Duchy of Mecklenburg-Schwerin	'57	539,258	197,622	69,215	266,837	1,198,450	157,522
Grand Duchy of Oldenburg	'52	279,637	—	—	219,843	295,322	87,336
Holland	'64	3,618,459	943,214	390,673	1,333,887	930,136	294,636
Belgium	'56	4,529,461	—	—	1,257,649	583,485	458,418
France	'62	37,386,313	5,781,465	8,415,895	14,197,360	33,281,592	5,246,403
Spain	'65	15,658,531	—	—	2,904,598	22,054,967	4,264,817
Austria	'63	36,267,648	6,353,086	7,904,030	14,257,116	16,964,236	8,151,608
Bavaria	'63	4,807,440	1,530,626	1,655,356	3,185,882	2,058,638	926,522
United States	'60	31,445,080	8,728,862	8,182,613	16,911,475	23,317,756	32,555,267